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Spin Determination of Ω^- in the Hyperon Beam

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ABSTRACT

We discuss the spin determination of Ω^- by the angular distribution of the ΛK^- system in the decay of the Ω^- produced along the beam direction.

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This note is based on a report presented at a Hyperon Beam Discussion Group meeting at NAL. We discuss the spin determination of the Ω^- produced in the forward direction by the angular distribution of the decay products ΛK^-. Much of the material presented here is already contained, explicitly or implicitly, in the paper of Byers and Fenster.[1] Our discussion makes use of a (perhaps) simpler procedure and is addressed exclusively to the spin determination of the Ω^-.

1. PRODUCTION DENSITY MATRIX

The Ω^- is produced in the forward direction. Then its density matrix $\rho(\Omega)$ is diagonal if the quantization axis is along the beam direction.

$$\rho(\Omega)_{mn} = \rho(\Omega)_m \, \delta_{mn} \quad . \tag{1}$$

We have the further property:

$$\rho(\Omega)_m = \rho(\Omega)_{-m} \tag{2}$$

which means that the Ω^- is unpolarized but may be aligned.

Equations (1) and (2) follow because

(a) An arbitrary density matrix for a particle of spin j at rest can be constructed from the components of the angular momentum operator $\vec{J}$. Consider the operator which has one non-zero matrix element, $P(m, n)$:

$$P(m, n)_{ij} = \delta_{mi} \, \delta_{nj} \quad .$$

A diagonal operator can be written as a polynomial in J_Z:

$$P(m, m) = a(J_Z - j) \ldots [J_Z - (m+1)] \, [J_Z - (m-1)] \ldots (J_Z + j)$$

$$a^{-1} = (m - j) \ldots [m - (m+1)] \, [m - (m-1)] \ldots (m + j) \, .$$

An off-diagonal operator can be written as $(m > n)$:

$$P(m, n) = b \, P(m, m) (J_+)^{m-n} \, P(n, n)$$

$$b^{-1} = \sqrt{j(j+1) - n(n+1)} \ldots \sqrt{j(j+1) - (m-1)m}$$

Since an arbitrary matrix can be written in the form

$$\rho = \Sigma \rho_{mn} P_{(m,n)} ,$$

it can be written as a polynomial in $\vec{J}$.

(b) The density matrix must transform as a scalar under rotations. Since it can only depend on the beam momentum $\vec{p}$, and since parity is conserved, it must be an even polynomial in $\vec{p} \cdot \vec{J}$:

$$\rho(\Omega_-) = a + b(\vec{p} \cdot \vec{J})^2 + c(p \cdot \vec{J})^4 + \ldots .$$

Equations (1) and (2) hold also in the case where the Ω is not produced forward but its angle relative to the beam is uniformly averaged.

2. DECAY MATRIX ELEMENT

The matrix element for the decay $\Omega^- \rightarrow \Lambda K^-$

$$\mathscr{A}_{sM} = \langle \Lambda(p,s) K^-(-p) | T | \Omega^-(JM) \rangle$$

may be expressed in the form:

$$\mathscr{A}_{sM} = \zeta_s^+ (\alpha + \beta \hat{p} \cdot \vec{\sigma}) C_{JM}(J - \tfrac{1}{2}, M-m; \tfrac{1}{2} m) \zeta_m Y_{J-\frac{1}{2}}^{M-m}(\hat{p}) \quad (3)$$

where ζ is a two-component spinor and α and β are constants. Expression (3) follows from the fact that the wave function describing spin $J = L + 1/2$ may be constructed by combining $J = 1/2$ and $J = L$ in the form of a two-component spinor with L (totally symmetrized) vector indices

$$\zeta_{i_1 \ldots i_L} \qquad i_j = 1, 2, 3 \qquad (4)$$

The auxillary conditions

$$\delta_{i_j i_k} \zeta_{i_1 \ldots i_j \ldots i_k \ldots i_L} = 0 \qquad \text{for any } j, k$$
$$\sigma_{i_k} \zeta_{i_1 \ldots i_k \ldots i_L} = 0 \qquad \text{for any } k \tag{5}$$

insure that the wave function (4) transforms as an object of total spin $L + 1/2$.

The total amplitude for $J \to (1/2, \vec{p}) + (0, -\vec{p})$ must be a scalar under rotations. Because of (5), only contraction of i_j with p_i will give a nonvanishing contribution which is proportional to the appropriate spherical harmonic. Furthermore, the expressions

$$C_{JM}(J-1/2, M-m; 1/2, m)\, \zeta_m\, Y^{M-m}_{J-1/2}(\hat{p})$$

and

$$\hat{p}\cdot\vec{\sigma}\, C_{JM}(J-1/2, M-m; 1/2, m)\, \zeta_m\, Y^{M-m}_{J-1/2}(\hat{p})$$

transform alike under the rotation group, but transform with opposite sign under space reflection. Thus, Eq. (3) is the most general form of the decay amplitude consistent with Lorentz invariance and parity violation. In Eq. (3), the Λ-spin dependence appears as a multiplicative factor

$$\alpha + \beta\, \vec{\sigma}\cdot\hat{p} \quad .$$

A consequence of the form (3) is that the longitudinal polarization of the Λ is independent of the decay angle; its measurement gives the relative parity violation but provides no additional information on the Ω^- spin.

3. DECAY DENSITY MATRIX

The density matrix describing the final state Λ is given by

$$\rho(\Lambda)_{ss'} = \mathscr{A}_{sM}(\hat{p})\,\rho_{MM'}(\Omega)\,\mathscr{A}^*_{M's'} = \zeta_s\,\rho(\Lambda)\zeta_{s'}$$

Using the expression (3) we may write

$$\rho(\Lambda) = (\alpha + \beta\,\vec{\sigma}\cdot\hat{p})\zeta_m\,\Lambda_{mm'}\,\zeta^\dagger_{m'}(\alpha^* + \beta^*\vec{\sigma}\cdot\hat{p}) \tag{6}$$

where

$$\Lambda_{mm'} = \sum_M \rho_M(\Omega^-)C_{JM}(J-1/2, M-m; 1/2, m)C_{JM}(J-1/2, M-m', 1/2, m')$$
$$\times\; Y^{M-m}_{J-1/2}(\Theta,\phi)\left[Y^{M-m'}_{J-1/2}(\Theta,\phi)\right]^* .$$

From relation (2) and the properties:

$$Y^m_L(\Theta,\phi) = (-)^m\left[Y^{-m}_L(\Theta,\phi)\right]^* = e^{im\phi}\left[Y^m_L(\Theta,0)\right]$$

$$C_{JM}(J-j, M-m; j, m) = C_{J,-M}(J-j, -M+m; j, -m),$$

we obtain:

$$\Lambda_{\frac{1}{2}\frac{1}{2}} = \Lambda_{-\frac{1}{2}-\frac{1}{2}} = \sum_{M>0} \rho_M\Big\{\left|C_{JM}(J-\tfrac{1}{2}, M-\tfrac{1}{2}; \tfrac{1}{2}, \tfrac{1}{2})\right|^2\left|Y^{M-\frac{1}{2}}_{J-\frac{1}{2}}(\Theta,0)\right|^2$$
$$+\left|C_{JM}(J-\tfrac{1}{2}, M+\tfrac{1}{2}; \tfrac{1}{2}, -\tfrac{1}{2})\right|^2\left|Y^{M+\frac{1}{2}}_{J-\frac{1}{2}}(\Theta,0)\right|^2\Big\} \tag{7}$$

and

$$\Lambda_{\frac{1}{2},-\frac{1}{2}} = \Lambda_{-\frac{1}{2},\frac{1}{2}} = \sum_{M>0} \rho_M C_{JM}(J-\tfrac{1}{2}, M-\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2})\,C_{JM}(J-\tfrac{1}{2}, M+\tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2})$$
$$\times\; Y^{M-\frac{1}{2}}_{J-\frac{1}{2}}(\Theta,0)\left[Y^{M+\frac{1}{2}}(\Theta,0)\right]^* e^{-i\phi}\left[1 + (-)^{2M}\right]$$
$$= 0$$

since M is half integer.

Therefore Λ is proportional to the unit matrix and

$$\zeta_m \Lambda_{mm} \zeta_m^\dagger = \Lambda_{\frac{1}{2}\frac{1}{2}} \sum_m \zeta_m \zeta_m^\dagger = \Lambda_{\frac{1}{2}\frac{1}{2}} \times I$$

so that $\rho(\Lambda)$ is determined by (6) to be:

$$\rho(\Lambda) = \Lambda_{\frac{1}{2}\frac{1}{2}} [\alpha^2 + \beta^2 + 2 \operatorname{Re} (\alpha\beta^*) \hat{p} \cdot \vec{\sigma}] \ .$$

For J=1/2 the decay is isotopic. Explicit calculation of $\Lambda_{\frac{1}{2}\frac{1}{2}}$ for J=3/2 gives an angular distribution of the form

$$d\Gamma(\Theta) \propto \frac{3}{8\pi} \left\{ [(\rho_{1/2}(\Omega) - \rho_{3/2}(\Omega)] \cos^2\Theta + \frac{1}{3} \rho_{1/2}(\Omega) + \rho_{3/2}] \right\}$$

which reduces to isotropy for an unaligned Ω beam:

$$\rho_{1/2} = \rho_{3/2} \ .$$

4. SPIN-DEPENDENT INEQUALITIES

The angular distribution is in general a polynomial in even powers of cos Θ. If the degree of the observed distribution is 2n, then:

$$J \geq n + 1/2 \ .$$

If the distribution is quadratic, is it possible to rule out spin higher than J = 3/2 ? Assume that the observed distribution is

$$W = a + b \cos^2 \Theta \qquad (8)$$

Then the mean value of the Legendre polynomial $P_2(\cos \Theta)$ is related to a and b by:

$$\frac{a}{b} = \frac{15 <P_2>}{2-5<P_2>} .$$

From the angular distribution (7) one can show[1] that for the decay of a particle of spin J the mean value of $P_L(\cos \Theta)$ is related to the density matrix elements by:

$$<P_L> = (2J+1) \Sigma \rho_m (-)^{m-\frac{1}{2}} \begin{pmatrix} J & J & L \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} J & J & L \\ m & -m & 0 \end{pmatrix} \quad (9)$$

Using the property

$$\sum_{L_{even}} (2L+1) \begin{pmatrix} J & J & L \\ n & -n & 0 \end{pmatrix} \begin{pmatrix} J & J & L \\ m & -m & 0 \end{pmatrix} = \frac{1}{2} \left\{ \delta_{mn} - \delta_{m-n} \right\} .$$

Equation (9) may be inverted to determine the density matrix elements:

$$(2J+1) \rho_n (-)^{n-\frac{1}{2}} = \sum_L \frac{\begin{pmatrix} J & J & L \\ m & -m & 0 \end{pmatrix}}{\begin{pmatrix} J & J & L \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}} (2L+1) <P_L> . \quad (10)$$

If a quadratic distribution is observed, then

$$<P_L> = 0 \quad \text{for} \quad L > 2 .$$

Explicit evaluation of (10) gives:

$$\rho_n = \frac{1}{2J+1} \left\{ 1 + 5<P_2> \frac{[3n^2 - J(J+1)]}{3/4 - J(J+1)} \right\} \quad (11)$$

The density matrix elements satisfy

$$\rho_n > 0 \qquad \Sigma \rho_n = 1$$

Then since

$$\rho_n = \rho_{-n}$$

we must have

$$0 \leq \rho_n \leq 1/2 .$$

Using this constraint in (11) for each value of n, one derives the inequalities:

$$-1/5 \leq \langle P_2 \rangle \leq \frac{2J+3}{20J}$$

or, equivalently, for a distribution of the form (8):

$$J > 1/2$$

$$-1 \leq \frac{b}{a} \leq \frac{2J+1}{2J-1} \leq 3 .$$

Thus an observed anisotropy in the range

$$2 \leq b/a \leq 3$$

implies $J = 3/2$; one in the range

$$5/3 \leq b/a \leq 2$$

implies $J = 3/2$ or $5/2$, etc. If the observed value is in the range

$$-1 \leq b/a \leq 1$$

it can only be inferred that $J > 1/2$.

REFERENCE

[1]N. Byers and S. Fenster, Phys. Rev. Letters 11, 52 (1963).